V Semester B.A./B.Sc. Examination, Nov./Dec. 2016
(Semester Scheme)
(Fresh) (CBCS) (2016-17 Onwards)
MATHEMATICS – V

Time : 3 Hours   Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions : **(5×2=10)**

1. a) In a ring (R, +, ·), prove that $(-a)\cdot(-b) = a\cdot b;\ \forall\ a, b \in R$.

   b) Define subring of a ring. Give an example.

   c) Give an example of

      i) Commutative ring without unity

      ii) Non-commutative ring with unity.

   d) Find the unit vector normal to the surface $xy^3z^2 = 4$ at (– 1, – 1, 2).

   e) Find the divergence of $\vec{F} = x^2y\hat{i} - 2xz\hat{j} + 2yz\hat{k}$.

   f) Prove that $E = e^{hD}$.

   g) Write Lagranges Interpolation Formula.

   h) Evaluate $\int_0^1 \frac{dx}{1+x}$ by Simpson's $\frac{3}{8}^{th}$ rule.

where

x	0	$\frac{1}{6}$	$\frac{2}{6}$	$\frac{3}{6}$	$\frac{4}{6}$	$\frac{5}{6}$	1
y = f (x)	1	0.8571	0.75	0.6667	0.6	0.5455	0.5

P.T.O.

PART – B

(2×10=20)

Answer **two full** questions.

2. a) Show that the necessary and sufficient conditions for a non-empty subset S of a ring R to be a subring of R are

   i) $a \in S, b \in S \Rightarrow a - b \in S$

   ii) $a \in S, b \in S \Rightarrow ab \in S$.

   b) Prove that every field is an Integral Domain.

OR

3. a) Show that the set of all matrices of the form $\left\{\begin{bmatrix} a & b \\ 0 & 0 \end{bmatrix} \middle/ a, b \in R\right\}$ is a non-commutative ring without unity with respect to addition and multiplication of matrices.

   b) Fill all the principal ideals of a ring $R = \{0, 1, 2, 3, 4, 5\}$ w.r.t. $+_6$ and $\times_6$.

4. a) Prove that $(Z_7, +_7, \times_7)$ is a commutative ring with unity. Is it a Integral Domain ?

   b) State and prove fundamental theorem of homomorphism.

OR

5. a) Prove that a commutative ring with unity is a field if it has no proper ideals.

   b) Prove that the mapping $f : (Z, +, \times) \to (2Z, +, *)$ where $a * b = \frac{ab}{2}$ defined by $f(x) = 2x, \forall x \in Z$ is an isomorphism.

PART – C

Answer **two full** questions.

(2×10=20)

6. a) Find the directional derivative of $\phi(x, y, z) = x^2 - 2y^2 + 4z^2$ at the point (1, 1, –1) in the direction of $2\hat{i} - \hat{j} + \hat{k}$.

   b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$, prove that

   i) $\nabla r^n = n\, r^{n-2}\, \vec{r}$

   ii) $\nabla\left(\frac{1}{r}\right) = \frac{-\vec{r}}{r^3}$.

OR

7. a) Show that the surfaces $4x^2y + z^3 = 4$ and $5x^2y - 2yz = 9x$ intersect orthogonally at the point (1, – 1, 2).

b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, show that $\nabla^2\left(\text{div}\left(\frac{\vec{r}}{r^2}\right)\right) = \frac{2}{r^4}$.

8. a) If $\vec{F}$ = grad $(x^3 + y^3 + z^3 - 3xyz)$, find div$\vec{F}$ and curl $\vec{F}$.

b) If ϕ is scalar point function and $\vec{F}$ is vector point function then

curl $(\phi\vec{F}) = \phi$ curl $\vec{F}$ + (grad ϕ) × $\vec{F}$.

OR

9. a) If $\vec{F} = (x + y + az)\hat{i} + (bx + 2y - z)\hat{j} + (x + cy + 2z)\hat{k}$, find a, b, c such that $\vec{F}$ is irrotational then find ϕ such that $\vec{F} = \nabla\phi$.

b) Prove that curl (curl $\vec{f}$) = grad (div $\vec{f}$) – $\nabla^2 \vec{f}$.

PART – D

Answer **two full** questions. **(2×10=20)**

10. a) Find a cubic polynomial which takes the following data :

x	0	1	2	3
f (x)	1	2	1	10

b) Find f(1.4) from the following data.

x	1	2	3	4	5
f (x)	1	8	27	64	125

using difference table.

OR

11. a) Evaluate $\Delta(e^{3x} \log 4x)$.

b) Find f(7.5) from the following data.

x	7	8	9	10
f (x)	3	1	1	9

using difference table.

12. a) Using Newton's divided difference formula find f(3) from the given data.

x	0	1	2	5
f (x)	2	3	12	147

b) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's $\frac{3}{8}^{th}$ rule.

OR

13. a) Using Lagranges interpolation formula find f(2) from the following data.

x	0	1	3	4
f (x)	5	6	50	105

b) Using Simpson's $\frac{1}{3}^{rd}$ rule, evaluate $\int_0^{0.6} e^{-x^2} dx$.